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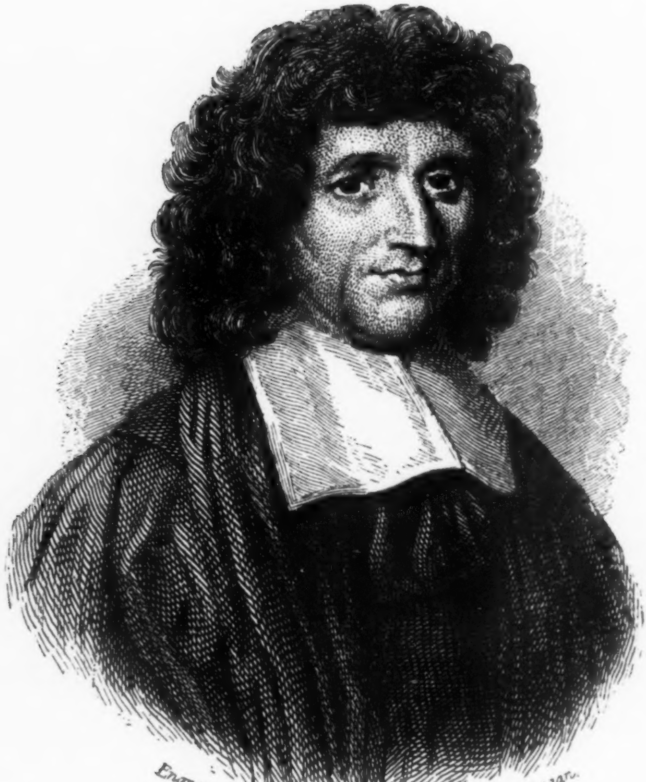

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REVD. ISAAC BARROW, D.D.

THE MATHEMATICS TEACHER

Volume XXV Number 8

Edited by William David Reeve

Formalism in Mathematical Teaching

By E. R. Hedrick
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1. *The nature of formalism.*—From the very wording of this title of mine—*formalism in mathematical teaching*—many among you may have inferred that I would decry formalism of every type in the teaching of mathematics. Were I to do so, however, I would needs defame mathematics altogether, for formalism is at the very basis of mathematical procedure. Precise formulation, abstraction, symbolisms, are the life-blood of mathematics, without which it would become anemic, sterile, worthless.

To defend all formalism on this ground, however, would be equally impossible. Mathematics may, and in fact does, employ formalisms; but formalism is not the end and aim of it: *Formalism* is, indeed, not in itself *mathematics.*

What are the aims of mathematics? What are our aims in teaching it? What are, indeed, the aims of education? In so far as formalism helps toward the realization of these aims, it is good and defensible. I shall call such formalism, *proper formalism.* Such proper formalism is necessary, and is one of the foundations upon which we build. When formalism passes beyond the bounds of necessary foundation, when it hinders rather than helps in the realization of our aims, it is no longer defensible, for it has in itself no virtues: such

needless formalism I shall call *improper formalism.* If I decry formalism, it will be this improper formalism, formalism for its own sake, the piling up of meaningless complexities for which no purpose is demonstrated. Just now, I need cite no examples, though I shall presently: every teacher knows how much there is in our texts and in our teaching that is *improper formalism* in this sense. No one of you will have failed either to note it or to deplore it. Yet we all continue, and our texts continue, to perpetuate it. Rather than to argue to you who know too well its *existence,* I would raise the question of what forces, what influences, are responsible for its persistence.

What is and what is not proper formalism, according to my definitions, depends upon our theory of the aims of education and, in particular, the aims of mathematical teaching. I shall assume that all of you are in accord with modern theories of education, and I shall not take your time to discuss in detail these aims; but I shall refer to many accepted principles of education in passing, in connection with ideas that depend upon them. As for mathematics, I shall assume a familiarity with the principles of the Report of the National Committee on the Reorganization of Secondary Mathematics, principles now widely adopted and copied into almost every recent discussion of mathematical teaching.

Examples are not lacking in the Report of the National Committee. Briefly I may mention as examples such instances as *factoring, ratio and proportion, limits and incommensurable quantities,* and *graphical representation of functions.* The discussion of any one of these would consume too much time, and I must finally refer you to the Report mentioned, or to some of the more or less accurate presentations which are based upon that Report. In factoring, we are advised to limit ourselves to the simplest cases: monomial factors, the difference of two squares, trinomials with simple real factors. How much of formalism is here omitted is quite obvious to you. In ratio and proportion, we are to dwell upon the meaning of a ratio as a fraction, and to emphasize the multitude of ratios, including the trigonometric ratios, that are used and are useful; but we are to omit the traditional so-called "theorems" on proportions, such as those on "alternation," "addition," "subtraction." We are to discuss measurement and commensurable quantities in that positive phase in which measurement is expressible in closed form, and we are to deal with limits and incommensurables only as they are approximated to by com-

mensurable cases. Graphical representation is to be emphasized in its relation to functional thinking rather than as a rack on which to hang successively more and more complex algebraic forms.

The demand that mathematical teaching shall have as its aim a training in thinking about relations between quantities, is at the basis of this; and this aim is accepted almost universally today, at least for public statements regarding our aims. It fits the modern educational demand that all education must strive to fit the student for life in the actual world: a world now as never before filled for every active person with quantitative relations, that is, with functional thinking. This statement means precisely that any active person, man or woman, in the modern world, must deal on every hand with quantities, and with relations between quantities. Who does not, accepts blindly the alleged expert advice of alleged experts. That this is true, in spite of the fact that it is said that *girls* need no algebra, is in sad evidence today: how many women today are staggering under some installment-paying, some financing of home or automobile, or some other "expert" plan involving partial payments, entered upon on "expert" assurance that the rate of interest was 6 per cent? Are there no such cases of which you know? Have you yourselves ever stopped to compute the actual interest charged? These women were once *girls,* and they may well have heard that algebra was useless, that quantities would never enter in their lives, except, perhaps, when "experts" would be ready to do the work for them, that to them vague generalities of the social sciences were more actual.

Does the mathematics of our schools prepare for life? Do students learn through it to control the quantitative situations of the world? I wonder whether the millions of women who are now in dire distress through interest rates on homes or loans or chattels, "expertly" rated at 6 per cent, really ranging between 17 per cent, and 50 per cent, how many such women, how many men, even vaguely connect this quantitative situation with *algebra?* Are they, though they studied algebra in our schools, still at the mercy of the "experts" who can make 20 per cent, look like 6 per cent? I wonder.

Shall education prepare for life? What are life's urgent problems? Shall we train students to meet these? Or is our slogan of "Education for Life" just a pretense? The quantitative situations of today are overwhelming us and are making miserable the lives of millions: taxes, bond issues, installment-buying, building and loan financing, graphi-

cal charts of financial and other trends, including the trends of population, relation of debased currency to prices and to wages; insurance, employment insurance, pensions; discounts, successive discounts, percentile profits on net and gross values. These things spell *life*—or *death*—to millions of men and women today. They are quantitative. They are calculable. The millions who are affected have, for the most part, studied enough mathematics to do the calculations, *if* mathematics has had as its aim to teach them to control the quantitative situations of life.

Well, has it? Do they know how? Do you think they might well? Do you believe in "education for life," or is that just propaganda, just a pretense to deceive a public that is deceived so easily and so often? Or perhaps you are of those who feel that mathematics—the relations between quantities—has no part in this "education for life"; it may be that you feel, as some seem to feel, that a knowledge of the campaigns of Cyrus or of Napoleon, a training in the nature of parliamentary government, or skill in fashioning wood-work, would be a solace to these desperate people; that that is "education for life."

Have you taught mathematics that would be more effective for present-day problems than would the history of art? or the nature of protoplasm? Very possibly not. Our schools announce boldly their intent to train for life; educators proclaim and all accept this principle. In mathematics, our National Committee, whose authority exceeds many fold that of any other body or individual, demands that we shape all our mathematics to give students control of quantitative situations; and every textbook that has been printed since gives its adherence to this principle, and abides by it at least as far as the end of the preface. Yet you know, as I know, that it is not done. Why? What counter influences exist? Why do we so often do those things that we ought not to do, and leave undone the things that we ought to do? Is there no health in us?

2. *Influences toward formalism.*—It is entirely possible that there are some present here who do not know that the mathematics of the secondary school—if so taught—would be sufficient, for the most part, for the calculation of all the problems that I mentioned above. I know personally teachers who have taught geometric progressions for years who are themselves ignorant of the fact that compound interest works that way. I know many teachers who have taught graphs for years who would not see the trick behind some of the statistical

charts issued by the million recently, by "experts" who know precisely what they are doing, on which the scale of ordinates of one graph is different from the scale of ordinates of the graph compared with it. To publish such a graph widely, as has been done, is vicious, immoral, and utterly damnable; and its success depends upon public ignorance. I might multiply such instances, and I might mention the myriad other connections of mathematics to life-problems; but the effect which I desire to produce is as clear now as it ever would be; and I wish to enquire rather about the influences which have been and are at work to make for formalisms and to thereby exclude such realities, from the teaching of mathematics.

Certain influences make, of course, only for proper formalism that is justifiable. Such is, for example, the very desirable tendency toward *generalization.* This is a fundamental mathematical tool, and, if it is not pressed too far, it is admirable. It forms one of the effective tools toward meeting the quantitative situations of life. To denote the dimensions of a figure by letters, and to get volumes, areas, and other dependent quantities expressed in terms of these dimensions; or to work problems in interest with *any* rate r, gives power over these and other similar quantities beyond the dreams of him who knows only arithmetic. The very possibility of generalization is held, indeed, by some modern writers on behaviourism, to differentiate between *man* and other animals, even more that the power to reason. To discover in himself this capability, is one of the greatest revelations possible in the life of any individual.

The formulation of rules, or, as one says in physics, "laws," in general terms, is another strong force toward proper formalism, if it is kept within bounds. It is strongly related, as are all such principles of action, to the idea of generalization which I have mentioned. In passing, let me remark that a given statement, or "law," is just as general, and just as "algebraic" in nature, whether we use letters to symbolize the primary quantities, or not. Thus Hooke's Law that the extension of a body under stress is proportional to the stress, or the Pythagorean theorem that the square of the hypotenuse is equal to the sum of the squares of the other two sides, is *algebraic* by virtue of its generality; it can be abbreviated by the use of letters, but it cannot be made any more "algebraic" than it is above. I have no patience with those who can see no algebra where there are no symbols; and I rather suspect that this inability to recognize algebra

without the customary symbols is responsible for some statements by men in education that mathematics is not used widely. We at least should recognize algebraic statements for their content rather than for their symbolism.

Such proper tendencies toward a proper formalism may be overdone, and may lead to excesses that are clearly improper. Thus, extensive use of literal exponents of complex form is a delight to the formalist. This is a clear instance of improper formalism. It has no place in public secondary instruction. Extensive treatment of logarithms to generalized bases other than 10 is another such that delights the formalist; while a mention of the general base may be made, extended discussion is another clear case of improper formalism. To demonstrate that it is wholly formal, you might sometime try this procedure on anyone who thinks such formalism proper. Since the amount of one dollar at compound interest for n years is the n-th power of $(1 + r)$, a table of compound interest is a table of anti-logarithms to the base $(1 + r)$. Does he who has emphasized greatly such generalized bases know that this is true? If not, to what end is his formalism?

A second influence toward formalism that persists in spite of all educational theory, is the residual belief, very strong in the minds of many, in the traditional disciplinary theory of education. That discipline of a different kind does exist and does function, I have not the slightest doubt. The dubious kind of discipline is that which sees good in mere slaving, purposeful or not purposeful. As Dooley has put it "making them *wurruk,* Mr. Hinnissy, particularly if it is something that is onplesant to thim." That gospel dies hard. Let me emphasize immediately that I myself favor *work;* I am not an advocate of soft education; but the work may as well be purposeful work, of which there is more than enough in mathematics. To dwell on a formal process only because it requires work is improper formalism, and has no place in public education. It is equally bad to *avoid* a topic because it requires work. To be proper, the topic must show a purpose; we have much that is purposeful, and that also requires hard work. Such we should choose and emphasize.

Allied with this evil form of discipline is the reliance on memory. Memorization of mathematical formulas is necessary to a degree, but anyone who has any experience knows that memorized mathematics is practically sure to be forgotten, unless it is restricted to very few

forms, and is based on thorough understanding. Trigonometry is the typical case of a subject often taught by memorization of hosts of formulas; I know well the results, for I see these students a little later in college. They have forgotten not only the hundreds of formulas they were supposed to have learned, but also even those few fundamental ones which they might have kept, had they been taught rationally. I have elsewhere compared their minds to an attic filled with an accumulation of articles, no one of which can be found when wanted: far better a few well selected articles of real worth, labeled, worked with, treasured, and in place when wanted. Such memorization of vast quantities of formulas or theorems, without emphasis or distinction, seems to me to be *not* useful, *not* mathematics, rather to be a phase of improper formalism. Surely a memorization that *is not to be permanent* is a waste of energy; teaching that results in it, or condones it, is unspeakably bad.

Quite a different tendency toward formalism is that due to certain types of examinations and to certain types of tests, many of the worst of which, unfortunately, have received much acclaim in certain educational quarters of late. Tests for attainment, tests for skills, tests for so-called "mastery" of a topic, are very likely to be almost purely formalized tests on ability to do a specific formal process. Questions that discover whether the student comprehends the meanings of what is done, or that test his ability to think on a quantitative situation, are practically absent. To my mind, the most curious thing in modern education is that some of the very men who condemn most strongly mathematics as a whole for its formalisms, apparently through oversight or ignorance have strongly favored such tests. In my opinion, there is at present no stronger force driving mathematics back toward the worst extremes of formalism. The spectacle presented by educational leaders who on the one hand decry formalism in mathematics, and on the other hand support such tests, is not a pleasant one. The extent to which this goes in cases is absolutely incredible. In a city that I shall not identify, except to say that it has over a quarter of a million inhabitants, I recently discussed these matters with a man who formerly taught mathematics, but who is now in charge of educational statistics. I felt that I had made progress with him in having him agree with me on some phases of the teaching of algebra and trigonometry; when he suddenly said: "But, Mr. Hedrick, you must not advocate the doing of anything in our classes which would inter-

fere with our testing program." If tests are to be placed in such exalted position, mathematics, in any true sense, is doomed.

Lack of knowledge, and inertia, are a last influence making toward formalism. Easier by far, of course, to assign the next fifteen problems, and to mark them. More of the same the next day. So goes, for example, factoring, when run to the extremes of improper formalism. Harder and requiring some energy, is it for the teacher to find illustrations of even the factoring of the difference of two squares. Do you hunt for such illustrations? Do you know any? Many? There are quite a lot of them. Indeed, if you are tempted to give a type-form in factoring, let this be your guide: ask yourself if you can find real illustrations of quantities that follow the type to be taught; if in your whole experience you can find none at all, either your own training is poor or else that *type of factoring represents improper formalism.*

3. *Training of teachers.*—Throughout this discussion, I, at least, have had in mind the question of the training of the teachers. What matters what we think, or here agree upon, if teachers widely through the whole country are not so trained that proper aims in education, on the one hand, and proper knowledge of the subject and its value, on the other hand, are known to them? I speak here in a city known for the excellence of its schools. Here, I may assume, the conditions of which I am about to speak do not exist. Or may I so assume? Have you sufficient data on the teachers in these magnificent high schools to be certain of your answers?

There is now in existence an International Commission on the Teaching of Mathematics, functioning through separate committees in every civilized country. David Eugene Smith, of Columbia University, is the chairman of the International body, and I have the honor to be the chairman of the American Committee, of which the other members are, Professor W. D. Reeve of Teachers College, Columbia, Miss (Dr.) Eva May Luse of the Iowa State Teachers College, Dr. Ben Frazier, of the Federal Bureau of Education, and Professor Ben Sueltz, State Normal School, Cortland, New York. This committee interlocks with the Federal Bureau through Dr. Frazier, and it will have the advantage of the great work now being conducted by that bureau in its Survey of the Training of Teachers, authorized by the last Congress. At the meetings in connection with the N.E.A. in Washington (February, 1932), I shall have opportunity to see results already secured by this survey throughout the entire country, and

I shall attend meetings of the Board of Consultants, of whose advisory board I am a member. Through this inspection, and through detailed work that we have planned, we shall know in great detail the actual statistics on the training of teachers throughout the country. Already, however, I know that the situation is not so good as we would all suppose. In one great northern state of excellent reputation, we have complete data for all of the high schools of reasonable size. In that state, of teachers who teach *only* mathematics in high school, the percentage of those who took any work whatever in college in mathematics is just 67 per cent, so that 33 per cent, of the full-time teachers of mathematics only in high schools of that state took not one course in mathematics in college. I should add that it is a fixed requirement for certification in that state that the teacher be a college graduate, so that each one did go to college. Such a situation, I fear, is rather the rule than the exception. In two other states in the North, the first and the second city, respectively, have such data, and it is fully as bad.

To inaugurate and to maintain a program of mathematics that shall avoid improper formalisms, we must obviously have teachers properly trained. Trained in the aims of education in general and of mathematics in particular; trained in mathematics itself so that they know what is important and vital, and what is time-serving and time-wasting.

Another principle of modern educational thought in which I believe firmly is the doctrine of interest. As you know, that doctrine holds that the educative process cannot go on efficiently in minds that are disinterested in the thing before them. The teacher must inspire interest in the subject, or it is indeed dead, fore-doomed to formalism. How may a teacher do this, if that teacher is himself uninterested? The picture that I conjure up, of a teacher himself not interesting, standing before a class of boys and girls, striving to make *them* interested where he is not, is about the most striking picture of hypocrisy of which I know. I have said before now, publicly, that I would prefer to have a teacher who had had no college training whatever, than to have one who, being in college, had carefully avoided taking a single hour of mathematical work. Some hope would remain, at least, for him who never went to college, that he had a real interest in mathematics. Your very presence here insures that you are not of that class, but if there be any teacher of mathematics in your schools, a college graduate, who avoided any mathematics whatever in college,

it is certain that the "interest" his pupils feel is a product of a proved hypocrisy. Formalism should run rampant where such a condition holds.

My hopes are hopes based on belief in the triumph of honesty. It *is* the best policy. In school and out of school. Do we announce principles only for propaganda and for deceit? Do we truly believe in "education for life"? In this our beloved field of mathematics—in this field that is and of a right ought to be the field of relationships between quantities—do we see life? Do we teach for life? Do we inspire our student with the love of it and with the sure means of meeting their own quantitative problems? If we *do not,* we are slowly but surely digging the grave of mathematics as a school subject; we are droning over dead formalisms that have no place in public schools; we are our own betrayers. If we *do,* mathematics may resume her rightful place in education, a place now often denied her, as the Queen of the Sciences.

A Study of Certain Mathematical Abilities in High School Physics*

By William Ray Carter
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VII. Statistical Interpretation of Results

The results from the five tests used in this study are presented in detail in the preceding part with but little comment or interpretation. A brief description of the distribution of first semester marks in physics is also included. It is now necessary to make further interpretations of these results in the light of the purposes of this study as they are stated in Part II.

The Validity of the Assumption upon Which this Study Was Based

As stated in Part II, the major problem of this study is based upon the assumption that the group of abilities having to do with the recognition of mathematical concepts has an influence upon performance in high school physics comparable to that of the group of computational abilities which have been studied by a number of other investigators.

The validity of this assumption is partly demonstrated in Part VI in connection with the description of the results from the tests employed. Evidence is presented there to show that the research test constructed to measure the student's ability to recognize mathematical concepts in physics situations functioned in a satisfactory way and gave results which show a higher correlation with the results from each of two mathematics tests than with either intelligence or reading ability, thus verifying the validity of the test. The validity of the fundamental assumption may be demonstrated in another way.

A comparison of the correlation between teachers' marks in physics and the ability to recognize mathematical concepts in physics situations and the correlation between teachers' marks and each of the other abilities measured should indicate the relative importance of the

* This is the third and last installment of an extensive study by Mr. Carter. The first and second installments appeared in the October and November issues of The Mathematics Teacher.—The Editor

former ability. When scores on the Carter test were correlated with first semester marks in physics, a coefficient of correlation of $.54 \pm .024$ was obtained. The coefficient of correlation between results from the Kilzer-Kirby test and teachers' marks was found to be $.53 \pm .024$.

These correlations indicate that there is a positive relationship between the ability to recognize mathematical concepts in physics materials, as here measured, and performance in physics, and that this relationship is at least as high as the relationship existing between performance in physics and the computational abilities measured by the Kilzer-Kirby test.

It was found that the coefficient of correlation between first semester marks in physics and intelligence, as measured by the Otis test, was $.53 \pm .024$. Likewise, the coefficient of correlation between marks and reading ability as measured by the Nelson-Denny test, was found to be $.45 \pm .027$.

These correlations suggest that the relationship between performance in physics and the ability to recognize mathematical concepts in physics material is as high as the relationship between performance and intelligence, as measured by the Otis test, and that the former relationship is considerably higher than the relationship between performance and the type of reading ability measured by the Nelson-Denny test. A summary of these correlations is given in Table XXI.

Of the correlations reported in Table XXI, the one between marks and reading ability is the lowest and is .09 lower than the correlation between marks and scores on the Carter Test for Mathematical Concepts in High School Physics. An application of the formula for the probable error of the difference between two coefficients of correlation and a reference to the appropriate probability table shows that there are 95 chances in 100 that this is a real difference greater than zero.[1]

These facts seem to indicate that the fundamental assumption upon which this study was based was valid and hence should lend some weight to the conclusions resulting from it.

The Recognition of Mathematical Concepts in High School Physics

One of the major purposes of this study, as stated in Part II, is the determination of the extent to which we may expect high school

[1] Garrett, H. E., *Statistics in Psychology and Education*. Longmans, Green, 1926. pp. 170-172 and p. 135.

physics students to be able to recognize the mathematical concepts involved in reading and understanding the text and other expositive materials in physics. The results from the research test constructed to measure this ability are given in Part VI. It is shown there that the median on this test is 50.6 and that the mean is 48.9. The middle

TABLE XXI

Correlations Between First Semester Marks in High School Physics and Scores on Each of the Tests Used in This Study

Test	Correlations with Marks
Carter	.54 ± .024
Butler	.52 ± .024
Kilzer-Kirby	.53 ± .024
Otis	.53 ± .024
Nelson-Denny	.45 ± .027

50 per cent are between 44.0 and 55.6, which is interpreted as indicating a rather high degree of ability in this respect. A comparison of the ranges of scores on the various tests will help in understanding the performance on the test for mathematical concepts in high school physics. The summary of the ranges on the five tests employed in this study is given in Table XXII.

A reference to Table XXII shows that there is a relatively smaller range on the Carter test than on any of the others, with the exception

TABLE XXII

Ranges of Scores on the Tests Employed in This Study

Test	Possible Score	Highest Score	Lowest Score	Range
Carter Test for Mathematical Concepts	66	65	22	43
Butler Test for Mathematical Concepts	63	63	27	36
Kilzer-Kirby (Computations)	66	64	16	48
Nelson-Denny (Reading)	172*	129	12	117
Otis Intelligence	72	69	12	57

* 172 is the highest possible score for college students. Only one per cent of high school seniors equal or exceed a score of 94.

of the Butler test. The lowest score on the Carter test is about one-third of the highest score, while on the Kilzer-Kirby test the lowest score is about one-fourth of the highest score. On the reading test and on the intelligence test, the lowest scores are, respectively, about one-thirteenth and one-sixth of the highest scores. These differences are doubtless due in part to differences in the difficulty of the tests.

The Otis test and the Nelson-Denny test were designed for use in colleges as well as in high schools, while the Carter test and the Kilzer-Kirby test were designed for high school students only. The differences in the ranges between the first two and the last two of these tests are not so significant if the medians and ranges on the first two are compared with the norms and usual ranges for high school juniors and seniors. It is shown in Part VI that the group is somewhat superior in performance on both the reading test and the intelligence test, and that this fact may partly account for the apparently high performance on the other two tests.

It appears, however, that even with proper allowance for other factors, there are reasonable grounds for the conclusion that the

TABLE XXIII

Coefficients of Variation for Each of the Five Tests Employed in This Study

Test	Coefficient of Variations*
Carter	16.96
Butler	15.3
Kilzer-Kirby	25.24
Otis	24.36
Nelson-Denny	32.2

* The coefficient of variation is a number showing the per cent that the standard deviation is of the average score. Since it gives less weight to extreme scores, it is more reliable for comparing dispersions on different tests than is the range.

high school physics students included in this study show a relatively high degree of ability to recognize mathematical concepts in physics situations, as that ability was measured by the Carter test. This conclusion may be further verified by a reference to the coefficients of variation for each of the tests. A summary of these is given in Table XXIII.

Apparently the subjects are less variable in the abilities measured by the Carter test and by the Butler test than in those measured by any of the other three tests. The greater variability on the reading test and on the intelligence test may be due in part to the fact that these latter tests were designed for both high school and college students, and in part to the greater number of items included, and the more varied reactions required on these tests. The difference in variability between the Carter test and the Kilzer-Kirby test cannot be explained as well on this basis, however, since both tests were designed

for high school students and were based upon actual physics situations. These tests have the same number of responses, but the Kilzer-Kirby test was apparently the more difficult and required a somewhat more varied type of response.

Although the facts thus far presented in this chapter lead to the conclusion that, on the average, high school physics students react rather well to the mathematical concepts in physics, it is probable that individual differences between students are great enough to be of considerable significance in the comprehension of physics text and expositive materials. The fact that the lowest student on the Carter test is only about one-third as efficient as the highest in reacting to mathematical concepts would obviously prove to be a handicap to him in a subject in which the mathematical elements are so frequently found. It is also possible that a smaller range in performance on the Carter test might be of more significance than a larger range on a more general test, such as an intelligence test or a reading test, since the former test is concerned with specific abilities directly involved in physics situations.

Relationships Between Computational Abilities and the Abilities Involved in the Recognition of Mathematical Concepts

As stated in Part II, one of the subsidiary aspects of this study is the determination of any relationships which may exist between the computational abilities necessary to the solution of physics problems and the abilities involved in the recognition of mathematical concepts in physics materials. Reference is made in Part III to the fact that the Kilzer-Kirby test is used to measure the computational abilities involved in the solution of physics problems.

In connection with a discussion of the validity of the Carter test in Part VI, it was shown that there is a higher relationship between the Carter test scores and the Kilzer-Kirby test scores than there is between the results from the Carter test and those from either the Otis test or the Nelson-Denny test. A further analysis of the coefficients of correlation between the results from the various tests employed in this study is necessary in order to establish the relationships between the abilities involved in the recognition of mathematical concepts in physics and those concerned with the solutions of physics problems. A summary of these correlations is presented in Table XXIV.

Reference to Table XXIV shows that the correlation of $.66 \pm .02$

between the results from the Carter test and those from the Kilzer-Kirby test is higher than the correlation found to exist between any other two variables, with the single exception of the correlation between the results from the Otis test and those from the Nelson-Denny test.

Further reference to Table XXIV shows that there is a higher correlation between the results from the Carter test and the type of reading ability measured by the Nelson-Denny test than there is between the results from the Kilzer-Kirby test and the same type of reading ability. This fact is shown by coefficients of correlation of .55 and .45, respectively. On the other hand, the Kilzer-Kirby test results show a higher correlation with intelligence, as measured by the Otis test, than do the Carter test results. This is shown by coefficients of correlation of .60 and .56, respectively. These differences are even more apparent when the partial correlation technique is used first to hold intelligence constant, and then to hold reading ability constant.

Table XXIV

A Summary of Correlations Between the Results from Four Tests Employed in This Study and Between Results from These Tests and First Semester Marks in Physics

	Kilzer-Kirby	Otis	Nelson-Denny	First Semester Marks
Carter	$.66 \pm .02$	$.56 \pm .02$	$.55 \pm .02$	$.54 \pm .02$
Kilzer-Kirby		$.60 \pm .02$	$.45 \pm .03$	$.53 \pm .02$
Otis			$.71 \pm .017$	$.53 \pm .02$
Nelson-Denny				$.45 \pm .03$

Table reads: The correlation between the Carter test scores and the Kilzer-Kirby test scores is $.66 \pm .02$; the correlation between the Carter test scores and the Otis test scores is $.56 \pm .02$; etc.

With intelligence held constant, the correlation beween the Carter test results and the Nelson-Denny test results is $.26 \pm .03$ and the correlation between the Kilzer-Kirby test results and the Nelson-Denny test results is $.04 \pm .03$. With reading ability held constant, the correlation between the Carter test results and the Otis test results is $.29 \pm .03$, while the correlation between the Kilzer-Kirby test results and the Otis test results is $.45 \pm .03$. These partial correlations are summarized in Table XXV.

The fact that the Carter test results show a higher correlation with reading ability than do the Kilzer-Kirby test results seems to be of some significance in view of the fact that the Carter test was designed to measure reactions to the mathematics involved in reading physics materials, while the Kilzer-Kirby test was constructed for the purpose of measuring the somewhat more varied reactions necessary to the solution of physics problems. The more varied type of response required on the Kilzer-Kirby test may be the reason for the fact that results from it show a higher correlation with intelligence than do results from the Carter test. The facts herein presented seem to indicate also that the Carter test and the Kilzer-Kirby test actually do measure different aspects of mathematical ability or different types of ability in mathematics.

TABLE XXV

First-Order Coefficients of Partial Correlation for Each of Two Mathematics Tests with Reading Ability and with Intelligence

Mathematics Tests	Correlations with Reading, Intelligence Constant	Correlations with Intelligence, Reading Constant
Carter	.26 ± .03	.29 ± .03
Kilzer-Kirby	.04 ± .03	.45 ± .03

Table reads: With intelligence held constant, the correlation between Carter test scores and reading test scores is .26 ± .03 as compared with a first-order partial of .04 ± .03 between reading test scores and Kilzer-Kirby scores, etc.

The relatively high correlation between the results from the Carter test and those from the Kilzer-Kirby test has been pointed out. This correlation is still relatively high, in comparison with the coefficients of correlation found in Table XXIV, even when intelligence is held constant. Using the partial correlation technique to hold intelligence constant, a first-order coefficient of partial correlation of .48±.03 between the results from these tests is obtained.

When both intelligence and reading ability are held constant, the coefficient of partial correlation between the Carter test results and the Kilzer-Kirby test results is .49±.03.

The foregoing comparisons indicate that, as the respective abilities are measured in this study, there is a relatively high degree of relationship between the ability to recognize the mathematical concepts involved in reading high school physics materials and the ability to

perform the computations necessary to the solution of problems in physics.

Ability to Recognize Mathematical Concepts Versus Computational Abilities in Relation to Success in Physics, as Measured by Teachers' Marks

As indicated in Part II, the second subsidiary aspect of our problem is concerned with finding the relative importance of (a) the ability to recognize mathematical concepts and (b) certain computational abilities in relation to success in high school physics, as measured by teachers' marks in physics. The method of obtaining uniformity in reporting marks and the nature of the distribution of marks thus obtained are discussed in detail at the end of Part VI. Certain evidence is presented there in support of the belief that this distribution of marks is satisfactory for our purposes.

(A) RELATIONSHIPS IN TERMS OF ZERO-ORDER COEFFICIENTS OF CORRELATION

A summary of zero-order correlations between the various test results, and between results on each of the tests and first semester marks in physics is given in Table XXIV. Reference to this table shows that the correlation between first semester marks in physics and results from the Carter test is $.54 \pm .02$. The correlation between first semester marks in physics and the Kilzer-Kirby test results is $.53 \pm .02$. The correlation between Otis scores and first semester marks in physics is $.53 \pm .02$ while the correlation between first semester marks and Nelson-Denny scores is $.45 \pm .03$.

From this summary of correlations, it appears that the relationship between performance in physics and the ability to recognize the mathematical concepts in physics is about the same as the relationship between performance in physics and the computational abilities involved in the solution of problems in physics. The results from both the Carter test and the Kilzer-Kirby test are found to have as high a relationship to marks as do scores on the intelligence test, and a considerably higher relationship to marks than that found between marks and reading ability, as measured by the Nelson-Denny Reading Test.

In order to show more clearly the relationships between first semester marks in physics and the respective abilities measured by the

Carter test and by the Kilzer-Kirby test, it is necessary to present the results of certain coefficients of partial correlation.

(B) RELATIONSHIPS IN TERMS OF COEFFICIENTS OF PARTIAL CORRELATION

When the partial correlation technique is used to hold intelligence constant, a first-order coefficient of partial correlation of .34±.03 is obtained between scores on the Carter test and first semester marks in physics. With intelligence constant, a partial correlation of .32±.03 is obtained between scores on the Kilzer-Kirby test and first semester marks in physics. With intelligence constant, the partial correlation between Nelson-Denny scores and first semester marks in physics is .12±.03. A summary of the relations between various test results and marks, in terms of zero-order coefficients of correlation

TABLE XXVI

A Summary of the Relations Between Various Test Results and First Semester Marks in Physics, in Terms of Zero-Order Correlations and First-Order Partials with Intelligence as the Constant Factor

Test	Correlations with first semester marks	
	(a) Zero-Order r's	(b) Partial r's, Intelligence Constant
Carter	.54 ± .02	.34 ± .03
Kilzer-Kirby	.53 ± .02	.32 ± .03
Nelson-Denny	.45 ± .03	.12 ± .03

Table reads: The correlation between Carter test scores and marks is .54 ± .02; and, with intelligence held constant, the correlation between Carter test scores and marks is .34 ± .03; etc.

and first-order coefficients of partial correlation with intelligence held constant, is given in Table XXVI.

These comparisons of zero-order correlations and first-order partial correlations with intelligence held constant suggest that there is still a positive relationship between first semester marks in physics and scores on either the Carter test or the Kilzer-Kirby test. These first-order coefficients of partial correlation are relatively high in comparison with the zero-order coefficients, and it is very probable that they are high enough to be of considerable significance in view of the fact that they are more than ten times as great as their probable errors. There seems to be a slightly higher relationship between marks and Carter test scores than between marks and Kilzer-Kirby test scores.

Some significance may also be attached to the fact that when intelligence is held constant there is a higher relationship between marks and scores on each of the mathematics tests than between marks and scores on the reading test.

When the partial correlation technique is used to hold reading ability constant, first-order partial coefficients of correlation of $.39 \pm .03$ between marks and Carter test scores and $.42 \pm .03$ between marks and Kilzer-Kirby test scores are obtained. These relations are summarized in Table XXVII.

TABLE XXVII

A Summary of First-Order Coefficients of Partial Correlation Between First Semester Marks in Physics and Test Scores, Reading Ability Being Held Constant

Test	Correlations with first semester marks (a) Zero-Order r's	(b) Partial r's, Reading Constant
Carter	$.54 \pm .02$	$.39 \pm .03$
Kilzer-Kirby	$.53 \pm .02$	$.42 \pm .03$

Table reads: The correlation between Carter test scores and marks is $.54 \pm .02$; and with reading ability held constant, this correlation becomes $.39 \pm .03$.

Inspection of Table XXVII shows that, in relation to the zero-order coefficients, there is still a relatively high correlation between first semester marks in physics and the scores on both the Carter test and the Kilzer-Kirby test when reading ability is held constant. It may be observed that with reading ability held constant the correlations between Kilzer-Kirby test scores and marks is higher than the correlation between Carter test scores and marks. Both of these correlations are found to be higher than the corresponding ones when intelligence was held constant (Table XXVI).

It has been pointed out earlier in this part that Otis test scores show a higher correlation with Kilzer-Kirby test scores than with scores from the Carter test, and that Nelson-Denny test scores show a higher correlation with Carter test scores than with scores from the Kilzer-Kirby test. The findings summarized in the two immediately preceding tables are consistent with this earlier conclusion.

When both intelligence and reading ability are held constant by means of the partial correlation techniques, the second-order coefficient of partial correlation is $.33 \pm .03$ between marks and Carter test scores as compared to a second-order partial of $.31 \pm .03$ between

marks and Kilzer-Kirby test scores. The relationship between Carter test scores and marks is thus found to be slightly higher than the relationship between Kilzer-Kirby test scores and marks when both intelligence and reading ability are held constant. These facts are given in Table XXVIII.

TABLE XXVIII

Correlations Between Marks and the Results from Two Tests When Both Intelligence and Reading Ability are Held Constant

Test	Correlations with Marks in Physics, Intelligence and Reading Constant
Carter	.33 ± .03
Kilzer-Kirby	.31 ± .03

Table reads: When both reading ability and intelligence are held constant, the correlation between Carter test results and marks is .33 ± .03; etc.

The second-order partials presented in Table XXVIII indicate a positive relationship between marks and the respective test results when conditions with respect to reading ability and intelligence are held constant.

Further evidence to support certain conclusions made in this section may be found by examining the third-order coefficients of partial correlation. These are summarized in Table XXIX.

A study of Table XXIX shows that with other conditions uniform, the Carter test scores show a slightly higher correlation with marks than is the case with respect to any of the other three tests employed in this study. Reading ability seems to have the least influence, in relation to the others, upon performance in physics as measured by first semester marks.

TABLE XXIX

Correlations of Various Test Scores with First Semester Marks, the Three Other Abilities Being Held Constant

Test	Third-Order Partials with Marks, Other Abilities Being Held Constant
Carter	.21 ± .03
Kilzer-Kirby	.19 ± .03
Otis	.18 ± .03
Nelson-Denny	.06 ± .03

The conclusions under this section of Chapter VII may be summarized by saying that the persistence of relatively high relation-

ships between first semester marks in high school physics and scores on the Carter test and on the Kilzer-Kirby test indicates a significant influence of these mathematical abilities upon performance in physics. The differences between the abilities measured by the Carter test and those measured by the Kilzer-Kirby test as influences upon performance in high school physics are small and perhaps slightly in favor of the Carter test. The importance of both of these mathematical abilities in relation to other factors involved in performance in high school physics is demonstrated consistently by the various statistical techniques presented in this part.

A Regression Equation for Predicting Performance in High School Physics

As an incidental outcome of this study, a regression equation was calculated in score form for the prediction of performance in high school physics upon the basis of scores from four of the tests employed in this study. This regression equation presents further evidence in support of certain conclusions contained in this chapter.

In the calculation of the regression equation, the procedure suggested by Garrett[2] was followed. The necessary zero-order coefficients of correlation are summarized in Table XXIV. The means and standard deviations for each test are given in various tables in Part VI in connection with the distribution of results from the various tests.

The following variables were involved in the calculation of the regression equation:

1. First semester marks in physics.
2. Recognition of mathematical concepts as measured by the Carter Test for Mathematical Concepts in High School Physics.
3. Computational abilities as measured by the Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics, Part I.
4. Intelligence as measured by the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B.
5. Reading ability as measured by the Nelson-Denny Reading Test for Colleges and Senior High Schools, Form A.

By using the procedure outlined by Garrett,[3] the following regres-

[2] Garrett, H. E., *Statistics in Psychology and Education,* Longmans, Green and Company, 1926. Chapter V, especially pp. 242-244.

[3] *Ibid.,* pp. 242-244.

sion equation in score form for the five foregoing variables was found:

$$X_1 = .074X_2 + .049X_3 + .052X_4 + .0073X_5 - 2.234$$

For the X_1 variable, marks were given a weight of one for each interval (M+, M—, M, etc.) used. The standard error of estimate for the foregoing equation was found to be 2.02, and $R_{1(2345)}$ was found to be .62.

This regression equation has certain limitations as far as any but the most general prediction is concerned. Thus, if an application of this equation to the scores made on the four tests by any individual indicated that he should make a grade of M (numerically represented by 6), the chances are about two to one that he will not make higher than S— (8) or lower than I+ (4).

The relation of this regression equation to the problem under consideration may be seen if each coefficient is multiplied by 20 in order to give a weight of approximately one to intelligence test scores (X_3). When this is done the equation becomes:

$$(20)X_1 = 1.48X_2 + .98X_3 + 1.04X_4 + .146X_5 - 44.7$$

In this equation the Carter test score (X_2) would have a weight of 1.48 (approximately 1.5) as compared to a weight of .98 (approximately 1.0) for the Kilzer-Kirby test score (X_3), a weight of 1.04 (approximately 1.0) for the Otis score (X_4), and a weight of .146 (approximately .15) for the Nelson-Denny score (X_5).

In other words, this regression equation indicates that for the purpose of predicting individual performance the Carter test score should be given a weight approximately 1.5 times as great as the weight for the Kilzer-Kirby test score or the Otis intelligence test score, and about ten times as great as the Nelson-Denny reading test score. The foregoing discussion applies only to the weight to be assigned to each *score* and does not refer to the relative contribution of the abilities measured to performance in physics.[4] In order to find the relative contribution to performance in physics made by (1) the ability to recognize mathematical concepts in physics situations, (2) the computational abilities included in the Kilzer-Kirby test, (3) intelligence, and (4) reading ability, as measured by our series of tests, it is necessary to use a special form of the regression equation

[4] Garrett, H. E., *Statistics in Psychology and Education*, Longmans, Green and Company, 1926, pp. 257-258.

as suggested by Garrett.[5] Following the procedure outlined by Garrett the special form of our regression equation becomes:

$$X_1 = .24X_2 + .21X_3 + .22X_4 + .06X_5 + K$$

This equation is interpreted as meaning that in so far as the ability to recognize mathematical concepts (X_2) as here measured, computational ability (X_3), intelligence (X_4), and reading ability (X_5) enter into performance in high school physics, as measured by first semester marks, they contribute with the relative weight of .24:.21:.22:.06. This may be interpreted for the purposes of our study as meaning that the first three of these abilities have about the same weight as influences upon performance in physics, and that each of these three abilities is considerably more important than the general type of reading ability measured by the Nelson-Denny test.

Conclusion

In Part VII various statistical techniques have been applied in the interpretation of the data presented in Part VI and the conclusions from this study that apply to the statement of various aspects of our problem as outlined in Part II have been presented in detail. These conclusions are summarized in the last part of this study.

VIII. General Conclusions and Summary

Part VII presents the conclusions which are more directly related to the major aspects of the problem under consideration in this investigation. These conclusions are presented there in some detail, but in a somewhat complicated form and in rather widely separated sections of that part. This was made necessary by the statistical techniques and procedures involved in establishing these conclusions. It is the purpose of this part to bring together these conclusions from the main body of the study and to give them an organized form.

Certain outcomes of this study relate themselves most readily to the field of psychology. These are listed as conclusions of a psychological significance. Because of their nature, other outcomes are listed as conclusions of a practical import. This division includes a reinterpretation of some of the psychological conclusions as well as those primarily of interest to teachers of high school mathematics and

[5] Garrett, H. E., *Statistics in Psychology and Education,* Longmans, Green and Company, 1926, pp. 256-258.

physics. Finally, there are a number of implications from this study which are not readily verifiable from our data. These are listed as suggestions for further study.

Conclusions of a Psychological Significance

Certain conclusions which are primarily of a psychological significance are among the more important outcomes of this study. Included in these are certain conclusions having to do with the nature of the abilities measured, the relationships between these abilities, individual differences in performance on various tests, and other conclusions which are related to the psychological analysis of subject matter in mathematics and physics. The following conclusions of a psychological nature seem to be justified by the evidence from this study:

1. The ability to recognize mathematical concepts occurring in the reading of high school physics seems to be an important factor in the comprehension of physics materials.

2. The ability to recognize the mathematical concepts involved in reading the text and other expositive materials in high school physics has been shown to exist in a way that can be measured objectively.

3. The ability to recognize mathematical concepts in high school physics, as measured in this study, seems to be primarily a mathematical ability in that correlations with other mathematical abilities are enough higher than the correlations with reading ability and with intelligence to indicate a probability of 98 to 99 chances in 100 of a significant different greater than zero in the respective coefficients of correlation.

4. The students employed in this study are decidedly less variable in performance on the Carter test and on the Butler test than on the other tests. This difference may be due in part to differences in the length of the tests and in the type and complexity of response required.

5. There is a higher correlation between reading ability and the ability to recognize mathematical concepts in high school physics than between reading ability and computational ability, as these abilities are measured by the tests employed in this study. Further investigation might show that the ability to recognize mathematical concepts is more or less closely related to certain recognition aspects of reading.

6. The importance of the ability to recognize the mathematical concepts in high school physics in relation to performance in physics has been shown by the fact that the correlation between performance in physics and this ability is slightly higher than the correlations between performance in physics and either intelligence or reading ability or the computational abilities measured by the Kilzer-Kirby test.

7. The importance of the relationships between performance in physics and the results from both the Carter test and the Kilzer-Kirby test suggests the advisability of including these types of abilities in the psychological analysis of subject matter in high school physics.

8. It is believed that the facts presented in this study indicate that the present controversy over the importance of mathematics in physics is due to a lack of a specific analysis of mathematical abilities, and that the techniques of analysis used in this study would offer a reliable solution of this problem.

9. The fact of individual differences is clearly emphasized in the range of performance upon each of the tests employed in this study.

10. Upon the basis of the results from the Carter Test for Mathematical Concepts in High School Physics, it seems that high school physics students, on the average, show a relatively high degree of performance with respect to the ability to recognize the mathematical concepts in high school physics which are included in this test.

11. The results from the tests employed in this study indicate that, with the exceptions of computational ability and variability of performance on some tests, the differences between boys and girls are not great enough to be considered very significant. The most marked difference between boys and girls is in computational ability, as measured by the Kilzer-Kirby test. Some significance may be attached to the fact that the boys are superior to the girls on this test as shown by a difference in medians of 6.5 and a difference in means of 3.4. The girls are slightly higher in reading ability and in intelligence, while the boys are a little higher on the other two mathematics tests. The least marked differences between boys and girls are in the abilities measured by the Carter test.

12. The boys were found to be about 14 per cent more variable than the girls with respect to scores on the intelligence test. They were more than 11 per cent more variable in reading ability and 1.3

per cent more variable on the Butler test. The girls were about 10 per cent more variable than the boys in computational ability, as measured by the Kilzer-Kirby test. The girls were also 1.8 per cent more variable than the boys on the Carter test.

Conclusions of a Practical Import

The following conclusions from this study seem to be primarily of interest to teachers of high school mathematics and physics:

1. The results from both the Carter test and the Kilzer-Kirby test, involving as they do test materials from specific situations in physics, are quite as significant as the results from the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B, in showing what a given student or group of students will probably be able to do with respect to general performance in physics.

2. In comparison with the results from the other tests employed in this study, the results from a general reading test such as the Nelson-Denny test are not very reliable for the purpose of predicting performance in physics. The wide range in reading ability shown to exist between physics students, however, may be of considerable importance with respect to the time which must be spent by different pupils in mastering a given assignment.

3. The range of scores on each of the tests employed in this study emphasizes the fact of individual differences even among students in advanced elective courses in the senior high school.

4. The relatively high median performance of the group on the Carter test indicates that high school physics students have a relatively high degree of ability to recognize in physics materials the mathematical concepts included in this test.

5. Physics teaching as now conducted, however, apparently does not contribute to any appreciable extent to the pupil's ability to react to the quantitative aspects of the subject, as this type of reaction was measured by the Carter test.

6. In view of the results presented in this study with reference to performance on the Carter test as influenced by previous training in mathematics, there seems to be some justification for the mathematical prerequisites which are now required by some schools for admission to the physics course.

7. On the other hand, the performance of many members of the group on certain fairly simple items in the two mathematics tests

which purport to measure mathematical abilities in physics situations, indicates that the fundamentals of mathematics are not thoroughly enough mastered by a large percentage of high school students to enable them to apply these fundamental processes in a field such as physics. This conclusion is in harmony with generally accepted principles of transfer of training, and should not be interpreted as implying a criticism of the schools which co-operated in this study or of the mathematics teachers in those schools. The consistency of the results from school to school seems to indicate that this lack of mastery of certain fundamental processes in mathematics is more or less general and that it may be due in part to curricular deficiencies such as a lack of emphasis on the applied aspects of mathematics.

8. The importance of the computational abilities measured by the Kilzer-Kirby test in relation to performance in physics seems to indicate that there is some justification for the inclusion in the physics course of the exercises of a computational nature which are now found in most physics textbooks. It is not within the scope of this study to determine the extent to which performance on exercises of this type entered into the determination of the marks assigned to the pupils included in this study.

9. The importance of the relationship between the ability to recognize mathematical concepts and performance in physics suggests that the inclusion of exercises of the type included in the Carter test might have important outcomes with respect to comprehension of physics materials. At least there is as much evidence in this study to justify the inclusion of exercises of this type as there is for the computational exercises commonly included.

10. The importance of the relationship between performance in high school physics and scores on both the Carter test and the Kilzer-Kirby test suggests the possibility of using these tests for the purpose of guidance of students who are about to enter the physics course. There is evidence that scores from these tests would be more reliable for this purpose than are the intelligence and reading test results which are used in many schools.

Suggestions for Further Study

The following problems for further study are suggested by the results from the investigation:

1. The relationships between the ability to recognize mathematical

concepts and the general type of reading ability measured by the Nelson-Denny Reading Test suggest the need for a study of the relationships between the ability to recognize mathematical concepts and various specific types of reading ability, especially of the recognition aspects of reading.

2. The relationships between the ability to recognize the mathematical concepts in high school mathematics and various computational abilities might profitably be studied in mathematics situations.

3. Certain relationships between the ability to recognize mathematical concepts in mathematics situations and performance in high school mathematics should be investigated.

4. The influence of various types of training and experience upon the development of the abilities concerned with the recognition of mathematical concepts should be studied.

5. The relatively high degree of relationship between the ability to recognize mathematical concepts in physics and performance in that subject indicates that an attempt to devise a plan for exercising this ability in connection with the regular work in physics might be both feasible and profitable.

6. A number of analytical tests might be constructed for diagnostic purposes in connection with a further study of the ability to recognize mathematical concepts either in physics situations or in mathematics situations.

7. In view of the important relationships shown in this study to exist between two types of mathematical ability and success in physics, it seems that a study of the transfer effects of mathematics training to physics situations might yield valuable results.

8. An investigation of the extent to which training in computational abilities contributes to the development of the ability to recognize mathematical concepts might be of value.

9. The relatively low relationship between performance in physics and the type of reading ability measured in this study suggests the need for studies of the relationship between performance in physics and various specific types of reading ability.

10. A study of methods by which high school mathematics training can be made to contribute more effectively to the development of the abilities concerned with the recognition of mathematical concepts seems to be needed.

An Idea that Paid

A PLAY DEVELOPED AND ACTED BY AN EIGHTH GRADE CLASS

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After approximately four weeks spent in a study of the arithmetic used in business by a proprietor of a store, by partners, and also by a stock company, this play was worked up in an eighth grade class. A review of the seventh grade work on Profit and Loss recalled the meaning of wholesale price, retail price, gross profit, overhead expense, net profit, and rate of income. The new work had to do with the forming of a partnership, the sharing of profits and of losses, and the determining of the rate of income; the forming of a stock company with emphasis upon the raising of capital by selling shares of common and preferred stock, the dividing of the net profit among the shareholders of each kind of stock, par as the original value of the stock, buying stock above and below par, and figuring the rate of income of the stockholder.

Every member in the class had an opportunity to make suggestions and to criticize the various scenes as they were worked out. The following leads were used as guides in developing the play:

1. A salable article is necessary to produce a profitable business, something new, time-saving, work-saving, and not too expensive.

Several suggestions were given but the class as a whole liked the idea of a "bed-maker" as described in the play. The pupils got a great deal of fun out of selecting names which would be suitable to the characters, and enjoyed talking over the possibilities of the workings of the "bed-maker."

2. An agreeable, conscientious, honorable business man increases business.

3. A man's rate of income tells more accurately his financial standing than does his income stated in dollars and cents.

4. There are some advantages and some disadvantages in a business partnership.

5. The formation of a stock company makes possible a larger capital

with which to do business and thus a larger income is derived if the business is profitable.

6. There are laws governing the forming of a company for the protection of the interests of all concerned.

Investigations were carried on by the pupils to get information regarding questions which naturally arose as the play developed. The value of dramatizing is greatly increased when the play is worked up by the performers. The choices of pupils chosen for the parts were approved by the class.

This play, with makeshift properties such as screens for partitions between rooms, the teacher's desk for the counter in the store, etc., was presented in the classroom before the class and a few visitors including the principal of the school. At the close of the presentation of the play a coincidence occurred which served to fix in the minds of the pupils the reality of stocks. Mr. Crowell, the principal of the school, stepped before the class, taking from his pocket a stock certificate. He had purchased it just that morning and had not yet placed it in his deposit-box at the bank. The bell for the passing of classes interrupted the discussion which was brought about by Mr. Crowell's apt questioning and business like explanations regarding the stock certificate.

AN IDEA THAT PAID

Time: The present.

Place: A store where electrical appliances are sold.

(MR. GOODSELLER, *a clerk, is arranging boxes behind the counter when the owner of the business enters*).

MR. GOODSELLER. Good morning, Mr. Thinkitout.

MR. THINKITOUT. Good morning, Goodseller.

MR. G. I lay awake last night thinking about your asking me to be your partner. I wasn't kept awake by worry but by excitement. I shall be so glad to accept your generous offer. I have saved up $5000. Will that do?

MR. T. That will be satisfactory. I wouldn't want everybody for a partner. We have worked together two years and in that time I have found you to be ambitious, honorable, and genial. You have drawn trade and have kept good customers. These customers will, no doubt, be interested in my invention, the bed-maker, and we shall bend our energies on selling it. (*He steps to the door of his office which is also seen by the audience, and addresses the stenographer,*

who is at her typewriter.) Good morning, Miss Stenotype. This is a fine morning isn't it? (*He hangs up his coat and hat.*) Miss Stenotype, please call up our lawyer, Mr. Knowalot, and ask him if he can come over here right away. Ask him to bring the necessary papers which should be used in forming a partnership.

MISS STENOTYPE. Very well, Mr. Thinkitout.

(A customer enters the store.)

MR. G. Good morning, Madam.

MRS. WANTOBUY. Good morning. I have come to find out about your bed-making device. A friend of mine has one and likes it very much.

MR. G. Yes, we have a wonderful device.

MRS. W. Will you explain it to me please?

MR. G. Certainly. The mechanism makes all of the beds in the house. (*He shows her a small keyboard of buttons to be pressed to operate the bed-maker.*) When installed, this keyboard of buttons is on the wall in the hall or living-room or wherever else you might wish to have it. Each pair of buttons represents a bed. When one button is pressed a bed is automatically made up.

MRS. W. And the red buttons?

MR. G. When fresh linen is to be put on the bed, it is laid on a chair at the foot of the bed. Then by pressing the red button the bed is made up fresh. In fact the bed when made up by this means looks perfect, much nicer than when made by hand.

MRS. W. Why is there a glass over the keyboard?

MR. G. To avoid an accidental touching of the buttons. If the bed is still occupied when the button is pressed it could cause a very uncomfortable experience for the occupant.

MRS. W. Would he get a shock?

MR. G. Not an electrical shock. Though this device is run by electricity, there is no current in the parts which are around the bed.

MRS. W. How long will it take to install the bed-maker?

MR. G. How many beds have you?

MRS. W. Three.

MR. G. About three to four hours.

MRS. W. What will it cost?

MR. G. A hundred dollars a bed plus fifty dollars for installing it, making it come to three hundred and fifty dollars.

MRS. W. What are your terms?

Mr. G. Ten per cent discount for cash, that is payment within thirty days, or fifty dollars down and easy monthly payments to extend over a year.

Mrs. W. That would mean more money, wouldn't it?

Mr. G. Yes, because there would be a small carrying charge.

Mrs. W. Of course you guarantee service?

Mr. G. Yes indeed. So far we have had no complaints.

Mrs. W. I'll pay cash. Please send a man out tomorrow to install it.

Mr. G. The name and address please?

Mrs. W. Mrs. I. Wantobuy, 3005 Modern Avenue, Uptodate Heights, Ohio.

Mr. G. Thank you Mrs. Wantobuy. That will cost you three hundred and fifty dollars less ten per cent or thirty-five dollars, which is three hundred and fifteen dollars.

(Exit Mrs. Wantobuy.)

(Enter Mr. Knowalot, the lawyer.)

Mr. G. Good morning, Mr. Knowalot. Come into the office. Mr. Thinkitout is in here.

(They go into the office. Each greets the others.)

Mr. Knowalot. Can this business be done in a hurry? I really should be at the Courthouse now.

Mr. T. Yes, Mr. Knowalot. We want to form a partnership and want it legal.

Mr. Knowalot. I see. A partnership. Well I guess you realize that each of you will be liable for the business debts incurred by the other? What do you value your business at?

Mr. T. $20,000.

Mr. K. What share will Mr. Goodseller hold?

Mr. T. $5000.

Mr. K. That means that Mr. Goodseller will receive ¼ of the profits and you ¾, that Mr. Goodseller will have to meet ¼ of the losses, if there should be any, and you ¾. Is that clear?

Both. Yes, Mr. Knowalot.

Mr. K. Is the business on a firm basis?

Mr. T. Yes. Thanks to Mr. Goodseller our sales have been phenomenal.

Mr. K. Had you ever thought of forming a stock company in order to do business on a larger scale?

MR. T. In time. At present we don't know enough moneyed men.

MR. K. Moneyed men are not the only ones who invest in stock. Besides I feel that you have invented a good salable article. It is well protected by being patented. I know quite a number of men and women who come to me for advice about investing their money. How would it do to have me arrange to have a few come here and talk the matter over?

MR. T. What do you think, Goodseller?

MR. G. Sounds good to me.

MR. T. Well, it sounds good to me, too. Can you do it soon?

MR. K. Yes. I'll get right at it today and we'll meet here next week. I'll give you a ring. I must rush along now. Good-day, gentlemen. (*He goes out*)

ACT II

Time: Thursday of the following week.
Place: Office of Mr. Thinkitout.

(*The telephone rings. Miss Stenotype answers it.*)

MISS S. Thinkitout office. (*Pause*) Oh, yes, Mr. Knowalot. Just a moment please. (*To Mr. Thinkitout.*) Mr. Knowalot wants to know if two o'clock today will suit you to have the meeting of the prospective investors here.

MR. T. Yes.—Tell him that will be fine.

MISS S. (*Talking over the phone*). Yes, Mr. Knowalot. That will be fine, Mr. Thinkitout says. Goodbye.

MR. T. It is now one-thirty. Get some chairs around the table. Did Mr. Knowalot say how many would be here?

MISS S. Yes. Three besides himself.

MR. T. We'll need seven chairs. Miss Stenotype, we'll need you.

MISS S. Very well, Mr. Thinkitout.

(*They arrange the room for the expected people and then busy themselves with office work. Enter Mr. Knowalot with Mr. Rollinmoney.*)

MR. K. Good afternoon, gentlemen. (*Greetings are exchanged.*) I would like you to meet Mr. Rollinmoney, Mr. Thinkitout, Mr. Goodseller. (*They in turn shake hands and express pleasure at meeting each other.*)

MR. K. (*As they take seats*). I am happy to bring you men together because Mr. Rollinmoney wants to know of some good way to invest his money, Mr. Thinkitout has a fine invention, and Mr. Goodseller

knows how to convince the public that they want what he has to sell. Two others are to meet us here. I expect them any minute. Ah! Here they are now. (*As Miss Game and Mr. Prosper enter.*) Good afternoon, Miss Game. Good afternoon, Mr. Prosper. (*Introduces each to those present. Mr. Thinkitout introduces Miss Stenotype to all.*) Now that we are all here, Mr. Thinkitout, will you please explain your invention?

MR. T. Yes. My invention is a bed-maker. I think it is not necessary to go into details about its construction as it would be difficult to follow. It is an electrical device controlled by a keyboard such as you see here. (*Holds up the keyboard.*) This set is for three beds. Each pair of buttons controls a bed. The keyboard may be installed any convenient place in the house. After the room has been well aired the housewife presses a button and the bed is made up. The red buttons are to be used when fresh linen is to be put on the bed.

MR. PROSPER. You don't say! Has it been well tried out?

MR. T. Yes. We have several installed right in this town. Mr. Goodseller got an order a few days ago to install one in Uptodate Heights.

MISS G. That certainly would be a blessing to us busy women! How does the bed look when it is made by the bed-maker?

MR. G. Perfect! Far better than when made by hand.

MR. R. What do you charge?

MR. T. One hundred dollars a bed and fifty dollars for installling it.

MR. P. Isn't that very cheap?

MR. T. If it is too expensive people will not buy. Furthermore at that price we make a good profit.

MR. P. What rate of profit?

MR. T. We have made as high as sixty per cent. But much of our profit will be used in advertising and paying high-priced traveling salesmen, and enlarging our offices, showrooms, and factory.

MR. R. Oh! Yes. Now that sounds like business.

MR. K. Mr. Thinkitout, what sum of money do you think would be a good working capital for a stock company to have for making and selling the bed-maker?

MR. T. I think $100,000 would be about right.

MR. R. (*Whistles*). $100,000! I'll have to have a lot faith in you gentlemen because I don't know very much about the workings of a stock company.

MR. K. You see, Mr. Rollinmoney, we shall have to get permission from the state to incorporate.

MR. R. To in—— what?

MR. K. To incorporate. By that I mean to become a legalized firm in the eyes of the state.

MISS G. Will you please tell us, Mr. Knowalot, just what must be done to form a company?

MR. K. Yes. To form a company there must be at least three persons two of whom are United States citizens. Right here we have seven. (*Looking around.*) You will want to be in this company won't you, Miss Stenotype?

MISS S. Yes indeed. I have some savings and know what a fine invention Mr. Thinkitout has and how well the business has been going.

MR. K. Well, to present the ideas I shall take for granted that we all shall become stockholders.

MR. R. Stockholders! That sounds familiar. I bought some stock the other day upon the advice of a stock-broker and a few days later he advised me to sell. I made some money doing that but I don't know how.

MR. K. We must get a permit from the State to become incorporated. We shall have to file articles of incorporation in Columbus. These articles will contain a brief account of the invention, the fact that it is patented, its use, the name of the inventor, the place of business, by-laws of the company, officers, directors, amount of the capital stock, and its par value. The name we give our company must not be similar to that of any other company.

MR. G. Well I am sure there is no other bed-maker company.

MR. K. Are there any questions?

MR. R. Yes. There are a lot in my mind but I'm ashamed to ask them.

MR. K. You need not be ashamed. What's on your mind?

MR. R. A little while ago you decided that $100,000 would be needed to carry on this business. Where is it coming from?

MR. K. The $100,000 will be divided into small portions which we call shares. Usually a share is one hundred dollars worth of stock. We could divide it this way: shares which cost one hundred dollars each as preferred stock, and shares which cost fifty dollars each as common stock.

MISS G. What is the difference between preferred and common stock?

Mr. K. Those who buy either kind receive a stock certificate showing the number of shares he owns and the par value of each. On the certificate of the preferred stock it also states the per cent of the par value to be received quarterly, semi-annually or annually by the owner of the stock. That is called the rate of dividend. The dividend rate is usually six, seven, or eight per cent. The common stockholder is not promised a stipulated dividend but is sure to receive something if the business is prosperous, and sometimes even receives a higher rate than that promised the preferred stockholder.

Mr. R. You mentioned "par" several times. What does that mean? I know what "par" in golf means.

Mr. K. "Par value" we shall think of as the original value of a share of stock. It is the amount that will be stated on our stock-certificates.

Miss G. How could a holder of common stock ever receive a higher dividend rate than the holder of preferred stock does?

Mr. K. With very careful bookkeeping the company determines what its profits are. At a directors' meeting a decision is made as to what should be done with the net profit. When the money promised the holders of the preferred stock is set aside for them there may be enough left to divide among the holders of the common stock. Divided profits are called dividends. Sometimes the profits are large enough to warrant a very large dividend for the shareholders. Those holding preferred stock get what is promised to them and the rest may go to the common stockholders.

Mr. P. Would it not be well for us to have the two kinds of stock? How about having the $100,000 divided in this way: Five hundred shares of preferred stock at one hundred dollars a share, that would mean $50,000, and one thousand shares of common stock at fifty dollars a share, that would mean the other $50,000?

Mr. T. That sounds very good to me. I have $20,000 and would like to buy one hundred shares of preferred stock and two hundred of common stock.

Mr. K. Miss Stenotype, will you please take this down?

Miss S. Yes, Mr. Knowalot, I am taking down everything.

Mr. G. I have five thousand dollars to invest. I should like to have twice as much common stock as preferred. That will mean, if my figures are correct twenty-five shares of preferred stock and fifty of common.

Mr. K. Right you are, Mr. Goodseller.

MISS S. I'll take five shares of preferred and ten of common. That is one thousand dollars.

MR. R. I'll invest $15,000 in one hundred shares of preferred and one hundred shares of common stock.

MR. P. I'll invest $10,000 by buying fifty shares of preferred and one hundred shares of common.

MISS GAME. I have only five thousand dollars which is not tied up. I'll buy twenty-five shares of preferred and fifty of common stock.

MR. K. I'll do the same. I'll buy twenty-five shares of preferred and fifty of the common stock. Miss Stenotype, I see that you have that all tabulated. Will you please total the number of shares of preferred stock and also the number of shares of common stock?

MISS S. Yes, Mr. Knowalot, there are 330 shares of the preferred stock which will amount to $33,000 and 560 shares of the common which will bring in $28,000. That means that we have raised $61,000.

MR. K. $61,000 from $100,000 means $39,000 worth of stock to sell. Mr. Thinkitout, could you go with me to Columbus next Monday? I have to go on other business and could combine the obtaining of a charter for this company with that job if you wish.

MR. T. Yes. I know of nothing that would interfere with my going.

MR. K. It is getting late and I must hurry away. Could we all meet here again tomorrow to elect officers and take necessary steps to have all in readiness for Mr. Thinkitout and me to present to the Secretary of State in applying for our charter? (*All signify their willingness as the curtain falls.*)

ACT III

Time: Two years later.

(*Mr. Prosper and Miss Game walking along the street.*)

MISS GAME. That was a fortunate day for me when Mr. Knowalot asked me to consider buying stock in the Thinkitout Bedmaker Company.

MR. PROSPER. For me too. I had my money in the bank and in scattered minor investments which didn't pay nearly so well.

MISS G. This year the common stock dividend rate is higher than that of the preferred stock just as Mr. Knowalot said might be the case.

MR. PROSPER. Here comes Mr. Rollinmoney.

MR. ROLLINMONEY. Well! Well! How do you do? (*They all ex-*

change greetings.) We all feel on top of the world don't we? I tell you that Thinkitout Bedmaker stock has paid me more than all my other investments have.

Mr. P. Have you heard that the directors want to enlarge the business and are authorized to sell more stock?

Miss G. Is that so? I shall want to buy some more.

Mr. R. I am sure it will sell easily. I have just come from the Stock Exchange where I found that the Thinkitout Bedmaker stock is selling fifteen points above par.

Mr. P. My Excelsior stock is way below par. It is at 82. I should like to get rid of it but must wait until it is worth more. I bought it at par.

Miss G. I believe that I shall go right over to Mr. Knowalot's office and ask him what I should do to get some more of the Thinkitout Bedmaker stock.

Mr. P. Didn't you get a letter from the Company? I got one today. They are giving their present stockholders an opportunity to buy at the original price, one hundred dollars for preferred and fifty dollars for common stock, before it will appear on the stock market, at its present market value.

Miss G. Is that so? I haven't been home since the mail delivery. Well! I shall certainly take advantage of that offer.

Mr. R. I wish I didn't have so much of my money tied up in stock that isn't paying.

(They bid each other farewell.)

CURTAIN

Scripta Mathematica

We take pleasure in announcing the first appearance of *Scripta Mathematica*, Vol. I, No. 1, for September, 1932, a quarterly journal edited by Jekuthiel Ginsburg and devoted to the Philosophy, History, and Expository Treatment of Mathematics. Associated editors of the quarterly are Raymond Clare Archibald, Cassius Jackson Keyser, Louis Charles Karpinski, Gina Loria, Lao Genevra Simons, and David Eugene Smith.

The magazine is published by Yeshiva College at Amsterdam Avenue and 186th Street, New York City. Subscription price $3.00 per year.—The Editor.

The Contribution of High School Teaching to Ineffective College Teaching*

By Joseph Seidlin, *Alfred University, Alfred, New York*

In a recent completed study on the teaching of college mathematics† I reached the almost inevitable conclusion that, barring exceptions, college teaching is generally ineffective in the sense that:

1. The methods of teaching procedure are chiefly a matter of setting tasks and quizzing.

2. Many accepted and readily acceptable laws in the psychology of learning are needlessly disregarded or violated.

3. The "examination" looms large as the chief incentive and source of motivation.

4. The "textbook" is magnified in importance thus allowing it to usurp disproportionate values.

5. Manipulative dexterity (Use of the formula and the formal reproduction of proof) is overly emphasized and stressed.

6. The "question" as an aid to teaching or learning is either neglected or abused.

7. Emphases on extraneous values, deadly prosaic routine, and other causes which need a great deal of detailed description and explanation.

In the process of gathering data on which the generalizations and conclusions of my study are based I had occasion to learn the complaints of the teachers of college mathematics. For the purpose of this paper I shall review merely such criticisms as were explicitly directed at the secondary school preparation of the college student. The quotations that follow form a diversified and fairly complete indictment of present day high school teaching.

1. "They [the students] come to us jaded, worn out, *weary* of mathematics."

2. "The most difficult task in teaching freshmen is this constant

* Presented at the meeting of Section 19 (Mathematics) of the New York Society of Experimental Research in Education, December 19, 1931.

† J. Seidlin, "A Critical Study of the Teaching of Elementary College Mathematics," Bureau of Publications, Teachers College, New York City, 1931.

pulling, tugging at the strings of undeveloped or starved affection for elementary mathematics."

3. "I give up teaching and become a mere drill master in the face of concentrated and unshakable prejudice [against mathematics] which seems to be the regulation marching equipment of these freshmen."

4. "They do not know the principles underlying the simplest operations. The greater number of our students become involved in the so-called advanced work because

a. They cannot perform such multiplications as

$$\frac{p^n}{q^n} \cdot q^{n-1}$$

b. They cannot add in such cases as

$$\frac{x + a_1}{p} + \frac{(x + a_1)(x + a_2)}{q}$$

c. They distrust the identity

$$-(-x^3 - x^2 - x - 1) \equiv x^3 + x^2 + x + 1$$

d. They have trouble when a fraction is to be raised to a power, *viz.:*

$$\left(\frac{p}{q}\right)^n = \frac{p^n}{q^n}$$

e. They are generally weak in the application of the laws of exponents (literal, fractional, negative).

f. They are extravagant in the use of the terms "transpose" and "cancel."

5. "They are frightened to incapacity by any new symbol or symbolic operation."

6. "Not one in ten is conscious of the positional scheme of our number system. That is why they experience such difficulty in working with detached coefficients."

7. "A study for the geologist or mineralogist: such perfect *cleavage* between operational mathematics on one hand and common sense on the other."

8. "Why are subscripted letters such anathema?"

9. "The most commonly expressed sentiment is 'I understand it but I can't express it.' "

10. "They lack ambition and initiative to generalize."

11. "They are happiest when a 'proof' (demonstration) is omitted."

12. " 'Method and Neatness'? Neither one nor the other can be readily detected."

Most of the above criticisms were made by older and experienced teachers in some of our most representative colleges and universities. These schools contain the chosen few of the relatively chosen few high school graduates who go to college.

The most serious indictment against the high school teacher of mathematics is that his product is "fed up" with mathematics.

Somehow under the high school teacher's tutelage the pupil—eager for demonstration at the age of 15—has become prejudiced against demonstration at the age of 18.

Primarily, the high school teacher is not blamed for failing to impart a sufficiently large number of facts to his pupils, but for apparently failing to inspire, or keep alive in, the pupil the more genuine values presumably obtainable through the study of mathematics. The implicit query raised in most instances is: "Of what conceivable value are more or less perfected schemes for imparting facts—facts soon forgotten—if both the foundation and the superstructure of these facts are so fatally impaired?"

Divers publications on the teaching of secondary school mathematics are replete with idealized objectives proposed to justify the time-honored position of algebra and geometry in the high school curriculum. Every good and sincere teacher of mathematics can and will justifiably defend the values and the realization of certain ideal objectives attainable in the study of elementary mathematics. These ideal objectives are variously described as:

1. Genuine and noble pleasure.
2. Feeding and encouraging the sense of wholesome curiosity.
3. Increased and more precious evaluation of contact with truth.
4. Clarity in thinking and expression.
5. Appreciation of the power and glory of logical proof.
6. Development of mental poise.
7. Development of the esthetic appreciation of form, structure, and symmetry.

Are we abandoning these aims? Is the teaching of mathematics becoming more prosaic, more immediately—however narrowly—practical, more painfully "functional"? Are the teachers of elementary

mathematics cluttering up the vision of their pupils so that some trees—it does not matter how few or many—obstruct the glory of the forest?

This is hardly a propitious time to borrow terminology from the business world (We hear so little, in these days of depression, of "selling" subject matter to our pupils), yet shall we not profit some by pausing a moment to take inventory of our so rapidly multiplying schemes and devices for imparting facts! Lest we forget the more genuine and lasting aims!

Kenneth Grahame

By James Gray from *The St. Paul Despatch* for July 12, 1932

It is difficult to believe that Kenneth Grahame who died the other day at his home in Pangbourne, England, was a man of 73. Though his books for children called *The Golden Age* and *The Wind in the Willows* were published before men now in their thirties were born, still it is a shock to know that their author has grown old in the meantime. One unreasonably expects the perennially youthful quality of the books to have kept the man eternally young.

Kenneth Grahame's very small literary output will probably always entitle him to a distinguished place in the history of English Literature. Beside *The Golden Age* and *Wind in the Willows*, there are only two other quite unpretentious books, *Pagan Papers* and *Dream Days*. But his literary charm and his understanding of the child's psychology are unique gifts which his enthusiastic cult of admirers will never permit to be quite forgotten. One generation that was brought up on *The Wind in the Willows* has already passed it on to a second, and that second is certain to remember and give it to a third.

What Sort of Minds Have Mathematicians?

It is a curious little fact of which, I should think, a really good theorizer might make an interesting theory that the authors of the two best loved classics for children written in English during recent years were in their more sober moments mathematicians.

Kenneth Grahame studied law and banking and for ten years at the height of his business career he was secretary of the Bank of England. His gift for mathematics is said to have been very brilliant and we are given to understand that he played a significant part in maintaining the reputation of the institution he served as one of the few staunch and unwavering things in the world.

And Lewis Carroll, the author of *Alice in Wonderland,* under his other name Charles L. Dodgson lectured on mathematics at Oxford.

Despite my personal prejudice in favor of the view taken by the Harvard student that "mathematics is a form of low cunning," I am beginning to believe with the other school that it is a beautiful mixture of philosophy, poetry and magic and its sacred mysteries can be grasped only by such serene and sensitive minds as those of Einstein and Kenneth Grahame.

Functional Geometry

By Charles Salkind, *Samuel J. Tilden High School, Brooklyn, New York*

Perhaps the coincidence of the appearence of the article by Barnet Rudman, "The Future Geometry," in the January, 1932, issue of this magazine with a similar article in preparation by the writer himself is not so strange after all. The big issue raised by Mr. Rudman is, or should be, uppermost in the minds of mathematics teachers. Training in reasoning is the premier of our objectives in the teaching of geometry. We justify the inclusion of this subject in the curriculum largely by the alleged or actual benefits to be derived from the ability to do postulational reasoning, the ability to select proper standards of logic, the ability to detect false conclusions, the ability to interpret results properly, and so forth. These are our trump cards in meeting the opposition of the Sneddens.

Yet, we appear to remain unconvinced by our own protestations. What has been offered for public consumption we ourselves have failed to assimilate. Why?

Mr. Rudman has done a valuable service in drawing our professional thought away from methodology toward content. Methodological features are of great importance, but their efficacy in economizing learning must not be overrated. Methodological technique, regardless of its degree of fineness, can never replace rich and varied associations as a means of establishing retention and easy recall. Permanence of learning and situational functioning are more likely to result from a conscious effort to relate one's teaching to other fields.

Nevertheless, it appears that Mr. Rudman is unfortunate in his emphasis and in his choice of illustrations. The proposition, "People should buy within their means even when buying on the installment plan," is not to be "concluded" in geometric style. Habit is the motivating force in such an instance, and habit is not changed by logical rigor. It is common knowledge and common experience that many a resolution is broken immediately after a successful intellectual process that it ought to be kept. Such propositions belong to the field of emotions, not the intellect.

The second proposition is an even more unfortunate illustration

because of an obvious moral tone. "To participate in gambling pools and outlawed lotteries is unpatriotic, unprofitable, and detrimental to a person's character." In the first place, what is the particular significance of the word *outlawed?* Is the moral turpitude of the gambler greater or less in this instance? Certainly, outlawry is not synonymous with unpatriotism. One who buys a ticket in a gambling pool may be indiscreet, antisocial; he may even be violating a law; but to say he is unpatriotic is to mistake a mole hill for a mountain.

Let us analyze the proof submitted in greater detail. Reason one is unsatisfactory. A citation of the statutes would have been better. Statement five is a paraphrase of statement four; with reason five far better than reason four. If statement six, especially the second half, obtained always, then countries like England and places like Havana, Monte Carlo, must be populated by only antisocial beings. This conclusion is a patent absurdity. Statement seven is a bit weak, for one may buy a lottery ticket and not spend the rest of his life day-dreaming and creating false illusions. Mr. Rudman should not overlook the possibility that in his classroom may be the son or daughter of a stock exchange broker.

The emphasis on "Life situations," it seems to me, should be not so much on the *form* of a geometric proposition. Factors such as necessary and sufficient conditions, plausibility of the given, relevancy of the given data, reliability of the conclusion, interpretation of the conclusion, sufficiency or insufficiency of the data—these are the elements of transfer that need stressing. Along with these the inculcation of certain habits and attitudes should be attempted. Suspended judgment, the king of them all, should be given royal attention. The feasibility or plausibility of the conclusion is a close second, and so forth.

In spite of these minor discordances Mr. Rudman's major chord is true. And towards the popularization of this tune we ought all to dedicate ourselves. In our periodic examinations, in our city-wide and state-wide examinations, we ought to request the inclusion of this type of material. Questions on this material can be inserted in the New York State Regents Examination in both the first and second parts. A few illustrations follow:

(1) A political candidate advertises that during his incumbency more schools were built than under his predecessor. You are justified in concluding that: (a) he is the sole cause of the larger number

of schools; (b) he had nothing to do with their increase; (c) you cannot know definitely without further information.

(2) What are the practical limitations to proving propositions in life by the method of "reducing to an absurdity"?

(3) A rectangular frame may be made rigid by adding another bar, (a) parallel to the short sides, (b) parallel to the long sides, (c) as a diagonal.

(4) Arguing in a circle means, (a) symmetrical reasoning, (b) always changing your mind, (c) coming back to the starting point.

(5) A speedometer on an automobile is intended for use with a tire of definite dimensions. Do you think that using oversize tires will make the speedometer read, (a) slower, (b) faster, (c) the same?

(6) Providing you can use either kind, which would you prefer to buy: (a) a six-inch pie selling for twenty cents, or, (b) a ten inch pie selling for forty cents?

In educational work particularly, because its scientific nature is incompletely established, we must guard against confusing the wishbone with the backbone. There are so many reforms we think ought to be instituted; many seem so reasonable and desirable when considered subjectively, that we fail to evaluate them critically, objectively, experimentally. Let us not be too sanguine. Let us seek much but be content with little. But, above all let us seek.

National Council Meeting!

Time—Friday and Saturday, February 24 and 25, 1933.

Place—Hotel Nicollet, Minneapolis, Minn.

Watch for the interesting program which President Betz has arranged and make your reservations at the Nicollet now. See the October issue of THE MATHEMATICS TEACHER (News Notes) for prices.

Isaac Barrow

1630—1677

ISAAC BARROW has been described as "an eminent mathematician and classical scholar, and one of the greatest of the great Anglican divines and preachers of the Caroline period."* His father was a linen draper in London—a man of royalist sympathies who followed the English court to Paris during the period of the Commonwealth and Protectorate (1649-1660). For a time, Barrow was a pupil at Charterhouse in London. His career there may be judged from his father's statement that "if it should please the good Lord to take one of his children, he could best spare Isaac." It is not surprising that his son was sent to another school, the one chosen being Felstead in Essex.

In 1644, Barrow entered Cambridge. He took his A.B. in 1648 and was made a fellow the following year. His interests were varied. He considered studying medicine, but gave that up for theology. In his estimation, a theologian needs to know chronology, and this necessitates astronomy, and this in turn implies mathematics. A knowledge of the classics was naturally assumed and Barrow's mastery of Latin and Greek may be guessed from the fact that in 1654, he was suggested by a former teacher as candidate for a professorship of Greek in Cambridge. A man whose influence was greater received the appointment.

In 1655, Barrow decided to travel, his decision was perhaps not wholly voluntary for his independent views may have brought him into disfavor with the authorities. He was obliged to sell his library to finance the trip. He spent some four years studying in Florence and in Constantinople and visiting Smyrna, Germany, and Holland. On the trip across the Mediterranean the ship was attacked by pirates and Barrow played his part in fighting them off. He later wrote verses in Latin to celebrate the event. By the time of his return to England, Barrow had published an edition of Euclid's *Elements* in Latin (1655) and an edition of Euclid's *Data* also in Latin (1657). It was this edition of Euclid's *Elements* that Newton bought as a school boy and dismissed after a single reading as a "trifling book."

Barrow served for a time as professor of Greek at Cambridge. In 1662, he became professor of geometry at Gresham College in Lon-

* *Dictionary of National Biography.*

don, but in 1663-64 he resigned this position to accept the recently founded Lucasian professorship of geometry at Cambridge. His *Lectiones Opticae* (1669) were published as the first fruits of this professorship. This volume was followed by *Lectiones Opticae et Geometricae* (1670) and by editions of Archimedes, Apollonius, and Theodosius (1675). The preface to the *Optics* contains this statement:

> However, after that I had entered on the intention of publication, either seized with disgust, or avoiding the trouble to be undergone in making the necessary alterations, in order that I should not indeed put off the rewriting of the greater part of these things, as delicate mothers are wont, I committed to the foster care of friends, not unwillingly, my discarded child, to be led out and set forth as it might seem good to them. Of which, for I think it right that you should know them by name, Isaac Newton, a fellow of our college (a man of exceptional ability and remarkable skill) has revised the copy, warning me of many things to be corrected, and adding some things from his own work, which you will see annexed with praise here and there. The other (whom not undeservedly I will call the Mersenne of our race, born to carry through such essays as this, both of his own work and that of others) John Collins has attended to the publication, at much trouble to himself.

In 1669-70, Barrow resigned his professorship to devote himself to theology, and named as his successor his most notable student, Isaac Newton.

In 1670, Charles II made Barrow his chaplain but called him "an unfair preacher because he exhausted every topic and left no room for anything new to be said by any one who came after him." It is said that Barrow's sermons were built on the plan of a geometric proof—but the proof must have been fairly long for a sermon that was delivered only in part occupied some three and a half hours.

Two years later, the king appointed Barrow as Master of Trinity College, Cambridge, saying that he was giving the position to the best scholar in England.

Barrow's chief contribution to mathematics was his "differential triangle"—a right triangle whose sides were the portion of a curve intercepted between two points, the difference between their ordinates and the distance between the ordinates. When the points approach one another, the triangle becomes similar to that made by the ordinate, the tangent and the sub-tangent. For the details of this work and for the analytic equivalents of Barrow's theorems, the reader is referred to the translation by J. M. Child, published by the Open Court Publishing Company (1916) with the title *The Geometrical Lectures of Isaac Barrow.*

NEWS NOTES

IT WILL BE greatly appreciated if the secretaries of the groups affiliated with the National Council of Teachers of Mathematics will send memoranda of progress of meetings, list of officers for the current year, and any other items of interest to Vera Sanford, 2060 Stearns Road, Cleveland, Ohio.

The Association of Teachers of Mathematics in New England, having upwards of 575 members, has the following list of officers for 1932:

President, Alexander C. Ewen, Dean Academy, Franklin, Mass.

Vice-president, William C. Graustein, Harvard University.

Secretary, Harry B. Garland, Brown and Nichols School, Cambridge, Mass.

Treasurer, Harold B. Garland, High School of Commerce, Boston.

Council, P. R. Crosby, Pawtucket High School, Pawtucket, R.I.; Anna R. Liden, Brookline High School, Brookline, Mass.; Titus E. Mergendahl, Tufts College; Lena G. Perrigo, Memorial High School, Boston; Marion E. Stark, Wellesley College; Elmer B. Mode, Boston University.

The programs of three of the meetings of 1931-32 were:

Annual Meeting, December 5, 1931, Boston, Mass.: "A Rational Presentation of Subtraction in Elementary Algebra," Susie B. Farmer; "A High School Mathematics Club," Enor E. Lundin; "Mathematics in the Twentieth Century," D. J. Struik; "Two-Too," William R. Ransom.

Midwinter Meeting, March 12, 1932, Portland, Me.: "Time Saving Devices in Geometry," Ada Bell Kennan; "A Solar Eclipse," Harley R. Willard; "Pitfalls in Algebra," W. L. Vosburgh; "Check and Double Check," Titus E. Mergendahl.

Spring Meeting, May 7, Boston, Mass.: "Geometry with Hinges," A. Harry Wheeler; "The Cube a Pandora Box," George H. Selleck; "The Mathematics Recitation," Walter S. Downey; "Repeating Decimals," D. C. Widder.

The officers of the Mathematics Section of the Western Convention District (Pennsylvania) for 1931-32 are:

Chairman, J. A. Silver, South Hills High School, Pittsburgh.

Vice-chairman, J. C. Stuchel, David B. Oliver High School, Pittsburgh.

Secretary-treasurer, Bertha J. Kirkpatrick, Westinghouse High School, Pittsburgh.

At the annual meeting on April 16, Dr. F. C. Touton of Los Angeles spoke on the "Rôle of Purpose and Method in Teaching Secondary Mathematics." The discussion of this paper was led by Katherine M. McKinney, Dormont High School, Dormont, Pennsylvania, and John V. O'Connor, Schenley High School, Pittsburgh.

The Buffalo Mathematics Club has about 60 members. The officers for 1931-32 were:

Chairman, Alberta Wanenmacher, Hutchinson High School.

Secretary-treasurer, Mary I. Brown, Grover Cleveland High School.

The club has a committee at work on the study of "Individual Differences." Dr. Charles W. Watkeys of the University of Rochester spoke at one of the two meetings of the year on "The Significance of Non-Euclidean Geometry to Mathematics." At the other meeting, Mr. Betz spoke on "The Newer Psychology as Applied to the Teaching of Mathematics." At the same meeting, Mr. Paul Smith of East High School, Rochester, discussed "Calculus in the Senior High School."

The Mathematics Club of Rochester, New York, has had a continuous existence of twenty-five years. The officers for 1931-32 were:

President, C. W. Watkeys, University of Rochester.

Vice-president, M. C. Hathorn, Madison Junior High School.

Secretary-treasurer, Rachel Longworthy, Benjamin Franklin High School.

The year's program had included: "Points of Contact of Spheres in a Pile," Professor Carver of Cornell; "The Mathematics of a Business Statistician," Professor D. W. Gilbert of the University of Rochester; "Discovery of a New Mathematical Manuscript," Professor Karpinski of Michigan; "Directed Line Geometry," Mr. Kimball of the University of Rochester.

During the year 1931-32, Section 19, Mathematics, of the New York Society for the Experimental Study of Education held seven dinner meetings at the Men's Faculty Club of Columbia University with an average attendance of seventy-five at the dinner and of one hundred and thirty-two for the discussions which followed. *The officers for 1932-33 are:*

Chairman, Professor W. D. Reeve, Teachers College, Columbia University.

Assistant Chairman, Professor W. S. Schlauch, New York University.

Secretary, Alma Ekholm, Girls' Commercial High School, Brooklyn.

The following outline shows the program for the past year:

October 24, Professor W. D. Reeve, "The Teaching of Mathematics in Germany."

November 21, Professor Veblen, "A Modern Approach to Geometry," and Mr. Urbane L. Barrett, "Lightning Calculation—Finding Cube Root."

December 19, Professor Seidlin, "Contributions of High School Teaching to Ineffective College Teaching," and Professor Vera Sanford, "Arithmetic and Its Perfection."

January 16, Mr. J. McCormach, "Finding Cube Root," and Professor Garabedian, "Mathematics in Relation to the Rest of Life."

February 27, Professor Hedrick, "Formalism in Mathematics Teaching," and Mr. J. McCormach, "Report of the Meeting of the National Council of Teachers of Mathematics held at Washington, D.C., February 1932."

March 26, Mr. J. T. Johnson, "A Technique in Teaching Mathematics to Provide Individual Differences," and Mr. C. C. Trueblood, "Teaching Mathematics to Large Size Classes."

April 30, Professor Longley, "The Function Idea in Algebra."

The meeting of the Mathematics Club of Greater St. Louis held its first meeting of the school year Saturday October 15 at the Educational Building, with the new president, Miss Meta

Eitzen of Beaumont High School, in charge.

The topic for discussion was the work done by the committee on "The Reorganization of Ninth Grade Mathematics." The report was discussed as follows:

1. The Committee and Its Work, Mr. L. V. Rader

2. Content of the Report, Mr. A. J. Schwartz

3. Standard Tests, Miss Anna Shapiro

4. Prognostic Tests, Mr. Arthur A. Glick

The interest of the mathematics teachers of St. Louis and environs in such timely topics as these was manifest by the large attendance and lively discussion which followed the presentation of the topics by the leaders.

The next meeting will be held December 3 at Washington University when the topics will be "The Recent Solar Eclipse," illustrated with pictures and "Problems on the Teaching of Geometry."

NEW BOOKS

Elementary Mathematical Analysis. By Mayme Irwin Logsdon, Volume I. Pages xiv+210. McGraw-Hill Book Co. Price $2.25.

Teachers of mathematics will be interested to see this new book which presents in a logical and interesting arrangement the fundamental elements of trigonometry, college algebra, analytical geometry and the calculus. The subjects are not presented traditionally, but are woven into a more or less general unit so that the proper interrelations are exhibited.

This is the first volume of a two-volume set which together are intended to form a satisfactory basis for further study in any standard course in the calculus. Incidentally the two volumes are intended to give those students who need it a good background in mathematics so that they may profitably pursue statistical studies in other fields, such as sociology, education, economics, psychology, and the like.

It is to be hoped that the appearance of such new books will ultimately lead to some general agreement as to the nature and method of treatment of such a course even in the secondary school.

An Arithmetic for Teachers. By William F. Roantree and Mary S. Taylor. Pages x + 523. Macmillan. Price $2.50.

This book is a revision of an earlier volume (1925) by the same authors. The purpose of the authors as set forth in the preface of this new book is to crystallize their experience since 1925—to improve the content and plan of organization in the light of present knowledge. To this end they have made several changes:

1. The chapters on "Factors and Multiples" and "Powers, Roots and Exponents" have been omitted and whatever merits therefrom are incorporated in connection with practical applications.

2. Addition and Subtraction formerly treated in separate chapters are now treated in one chapter because of their close relationship in teaching children. Multiplication and Division have been combined for the same reason.

3. Historical materials are placed at the beginning of a chapter.

4. The distinction between "Teachers' Knowledge" and "Methods of Teaching" has been sharpened.

Teachers giving courses on the teaching of arithmetic will wish to examine this new book.

Introduction to Trigonometry and Analytic Geometry. By Ernest Brown Skinner. Pages xi + 189. Macmillan. Price $1.50.

In this book the author attempts to accomplish two things. In the first place he tries to shorten the road to the calculus; secondly, he provides a course for the second half of the freshman year in college the purpose of which is to try to obtain the interest of the student who wishes to know something of the fundamental ideas of mathematics below the calculus.

The book has been prepared upon the premise that neither trigonometry nor analytic geometry alone constitute a full half year course for most students at this stage of their development. Moreover, the more difficult parts of both subjects may be better postponed until a student has acquired some knowledge of the calculus.

Differential and Integral Calculus. By John Haven Neeley and Joshua Irving Tracey. Pages viii + 490 Macmillan. Price $4.00.

This book is intended for use both in the liberal arts college and in engineering schools. It gives a comprehensive review of as much of analytical geometry as is required for success in understanding the calculus and recommends that this part be omitted where such work is not necessary in a given course. A feature of the book is to include the proofs which can be understood at this stage in the students' career, but plays the game squarely with the student by pointing out the places where assumptions are made.

Brief College Algebra. By W. L. Hart. Pages viii + 334. D. C. Heath and Co. Price $1.96.

The author has here set forth a review of the fundamental ideas of elementary algebra together with the outstanding ideas of the essentially collegiate parts of college algebra.

The book is organized in such a way as to be flexible. It is intended as a text either for those who have had two or three semesters of algebra previously. It may also be used by students of engineering or business.

The chapter on the mathematics of investment will be of interest to teachers who wish material of this nature prepared by one who is an authority in the field.

Mathematical Tables and Formulas. By Robert D. Carmichael and Edwin R. Smith. Pages viii + 269. Ginn and Co. Price $2.00.

These tables and formulas have been compiled by the authors for the use of students in mathematics courses or in other fields where numerical computation is required or which involve the understanding of processes based on that part of mathematics up to and including the calculus.

Part I contains five-place tables to be used in college algebra and trigonometry. Part II includes additional numerical tables which are often used in com-

putation but which are not generally accessible to students of mathematics in colleges. Part III contains many carefully chosen mathematical formulas for ready reference, together with tables of integrals and series.

Teachers will doubtless welcome a collection of tables chosen so as to facilitate both the needs of the inexperienced as well as the experienced computer.

Mathematics for Junior High Schools, Triangle Series. Book I, Grade 7; Book II, Grade 8; Book III, Grade 9. By Leo J. Brueckner, C. J. Anderson, G. O. Banting, Laura Farnam and Edith Woolsey. Pages xiv +370, for Books I and II and xiv + 456 for Book III. Prices $1.00 (Books I and II), $1.28 for Book III.

This new Junior High School series will be of interest to educators who are already familiar with the Triangle Arithmetics, by the same authors. Like the arithmetics, the Junior High School series was developed in the classroom, by the authors themselves and by teachers working under their supervision, and was completely tested and standardized with thousands of pupils before it was put into book form. The authorship includes arithmetic specialist, reading specialist, university authority, public school administrator and classroom teacher, so that every phase of the teaching of mathematics in the junior high school has received adequate attention.

The organization is intended to be so skillful that the material will not be in the teacher's way. The informational content, the suggested local applications, and the many provisions for individual differences, provide optional material of valuable sort. Organization is for unit teaching in the real sense of the word. Each chapter is a coherent unit composed of smaller units suitable for single lessons. Each unit carries the pupil one step forward to a definite goal.

Perhaps the most noteworthy single feature of the series, and one which is certainly not duplicated elsewhere, is the complete and systematic program, consisting of Survey Tests to segregate pupils according to needs; Comprehensive Diagnostic Tests in each process and in problem-solving, with cross references to remedial exercises; Practice Exercises to adapt the remedial instruction to individual needs; Problem Scales to measure ability in problem solving; and Practice Tests for review and maintenance of skills.

The pupil is led to think about quantitative aspects of life, such as the mathematics of the consumer in buying and selling; the mathematics of the home in budgets, accounts, bills, installment buying, insurance, saving and investing; and the mathematics of business and industry in simple units built around typical industries. Thus he should develop a true appreciation of the utility of number and its importance in social and occupational problems.

The format is striking. The books are sturdily bound in handsome cloth—a different color for each grade. Illustrations are abundant and interesting. They are not used merely for "flash" but are directly applicable to the text matter. There are colored illustrations for variety, but not so many that the pupil feels that the books are babyish. There is a great variety of modern business forms, printed in color and on safety paper—a feature not previously noticed in any other text. The type is large and clear, and pages are well-arranged—each page usually a complete unit. All in all the books seem inviting.